# Interpretation: The Final *Spatial* Frontier

Supplementary Files

## Overview

This document provides arguments discussed in—though not included in—the manuscript.

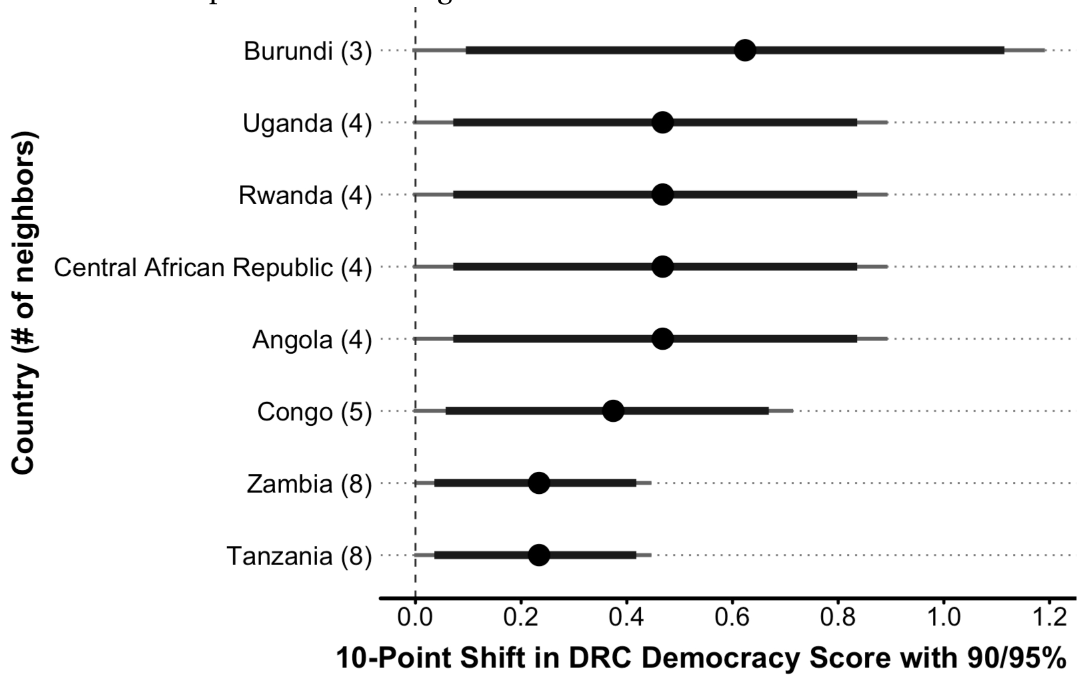
## SAR quantity of interest: effects of an unmodeled shock in the outcome

Another quantity of interest from the SAR model explores how an unmodeled shock to the outcome in observation i influences its neighbors through Equation 15 in the manuscript. In Figure A.1 we demonstrate how a 10-point increase in the Freedom House score for the Democratic Republic of the Congo (DRC) in 1994 influences its first-order neighbors' democracy levels. All eight of its contiguous neighbors experience a positive spillover (statistically significant at the 90% confidence level), and the effects range from +0.23 for Tanzania and Zambia to +0.62 for Burundi. Though each country in Figure A.1 is contiguous, the size of the effects vary as a result of the number of neighbors in the row-standardized weights matrix. For example, Burundi experiences over twice the spillover that Zambia experiences, because the DRC is one of three neighbors of Burundi, but one of eight neighbors for Zambia. Since spatial spillovers influence each of the eight neighbors equally (due to the row-standardization), the positive spillover as a result of the DRC's democratic improvement will be smaller for those observations with more neighbors.

## Model comparison and the coefficient interpretation approach

For example, consider the analysis of welfare state size during the Cold War by Obinger and Schmitt (2011) as an example of the perils of the *coefficient interpretation approach*. The au-

Figure A.1: Spatial Diffusion Patterns Following a 10-point increase in the Freedom House score for the Democratic Republic of the Congo




thors theorize that the reason for the drastic expansion of the welfare state in the Cold War was the regime competition between those in the capitalist West and those in the communist Soviet bloc (247). The authors theorize that three sets of connections between capitalist and communist states determine the degree of competition: all connections weighted equally, geographic proximity (inverse distance between capitals), and economic size. They then estimate three SAR models and three SLX models (with the spatial lags temporally lagged one year). Their interpretation is limited to pointing out that the spatial coefficients are positive, as expected, and that the coefficients for the control variables—such as economic growth and economic wealth—are also signed in the expected direction (263). Unfortunately, this approach provides no sense of the average effects due to positive spatial dependence, the average total effects, or *which* countries' connections are most meaningful. In short, the *coefficient interpretation approach* neglects a litany of theoretically-interesting inferences.

The authors estimate three SAR models and three SLX models (each time changing the specification of a row-standardized weights matrix). They compare the coefficients across the SAR and SLX models to conclude that the SLX models have a "more substantial effect for the regime competition variable" (262). The authors then caution in a footnote immediately following that comparison that "due to different estimation techniques the results cannot be strictly compared. They can only indicate tendencies about the relative importance of particular effects" (262). The authors are correct that in the *coefficient interpretation approach* comparing the coefficients is a fool's errand. On the other hand, the *general approach* allows scholars to directly compare the effect sizes across different spatial model specifications with the use of the partial derivative matrix.

## Row-standardization illustration

A common strategy is to row-standardize the weights matrix. However, the row-standardization process forces the average total effects to be the same; each observation is equally influenced by other influences. Recall the example in the manuscript of four observations that are located at points 0.5, 4.5, 5.5 and 7 on a single dimension. In the un-row-standardized version, the elements of **W** capture the inverse absolute distance (**W**), or $\frac{1}{\text{abs}(p_a - p_b)}$. Row-standardizing the matrix involves dividing each element by the row total. The resulting partial derivatives matrix and total effects are shown in Equation 1.

$$\mathbf{W}=\begin{pmatrix} 0 & .41 & .33 & .25 \\ .15 & 0 & .61 & .24 \\ .11 & .54 & 0 & .36 \\ .13 & .33 & .55 & 0 \end{pmatrix}, \quad (\mathbf{I}_N - \rho\mathbf{W})^{-1}\beta=\begin{pmatrix} .501 & .022 & .019 & .014 \\ .008 & .502 & .031 & .014 \\ .006 & .028 & .503 & .019 \\ .007 & .018 & .029 & .502 \end{pmatrix}, \quad TE=\begin{pmatrix} .556 \\ .556 \\ .556 \\ .556 \end{pmatrix} \tag{1}$$

The result is that each observation has the same total effects.

## Extensions

There are important extensions of the approach we outline here. The partial derivatives approach sheds light on how the effects of covariates change as a function of spatial location, or the degree of spatial heterogeneity. Other approaches can deal with spatial heterogeneity, either in a discrete (geographic regression discontinuity design, see Keele and Titiunik 2014 and Keele et al. 2015 for examples) or continuous manner (geographically weighted regression, see Brunsdon et al. 1996, Darmofal 2008, and Darmofal 2015 for examples). All of these approaches reveal any underlying spatial heterogeneity in effects, whether it is through observation-specific parameters (GWR) or a global parameter that is weighted by spatial location (partial derivatives). The latter approach is especially useful in the presence of continuous spatial heterogeneity and has applications primarily in geographic-based connectivity, although it need not be limited in that way.